Redistributive Allocation Mechanisms

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*Authors' names are in certified random order.

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Why not use a market mechanism when we know that it leads to efficient outcomes? One answer: **Redistributive concerns**

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  - ▶ the interaction between social preferences and observability of agents' characteristics;
  - ▶ how the revenue generated by the mechanism is used;
  - ▶ whether the good is essential or not.

Prior literature

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- **Dworczak $^{(r)}$ Kominers $^{(r)}$ Akbarpour (2020)**: two-sided market but "maximally simplified" (no heterogeneous quality, no observable information, no restrictions on lump-sum transfers)

- **Price control and public provision of goods:** Besley and Coate (1991), Blackorby and Donaldson (1988), Viscusi, Harrington, and Vernon (2005), Gahvari and Mattos (2007), Bulow and Klemperer (2012)

- **Optimal taxation:** Diamond and Mirrlees (1971), Atkinson and Stiglitz (1976), Akerlof (1978), Saez and Stantcheva (2016)

- **Auctions with budget-constrained bidders**: Che and Gale (1998), Fernandez and Gali (1999); Che, Gale, and Kim (2012); Pai and Vohra (2014); Kotowski (2017)

- **Costly-screening/ money-burning literature:** McAfee and McMillan (1992), Damiano and Li (2007), Hartline and Roughgarden (2008), Hoppe, Moldovanu, and Sela (2009), Condorelli (2012), Chakravarty and Kaplan (2013), Olszewski and Siegel (2019)

- **Methods (generalized ironing):** Muir and Loertscher (2020), Ashlagi, Monachou, and Nikzad (2020), Kleiner, Moldovanu, and Strack (2020), Kang (2020).

Outline

Model

Model

- A designer chooses a mechanism to allocate a unit mass of objects to a unit mass of agents.

- Each object has quality $q \in [0, 1]$, q is distributed acc. to cdf F.

- Each agent is characterized by a type (i, r, λ), where:

- i is an observable **label**, $i \in I$ (finite);

- r is an unobserved **willingness to pay** (for quality), $r \in \mathbb{R}_+$;

- λ is an unobserved **social welfare weight**, $\lambda \in \mathbb{R}_+$;

- The type distribution is known to the designer.

- If (i, r, λ) gets a good with quality q and pays t, her utility is $qr - t$, while her contribution to social welfare is $\lambda(qr - t)$

The designer has access to arbitrary (direct) allocation mechanisms (Γ, T) where $\Gamma(q|i, r, \lambda)$ is the probability that (i, r, λ) gets a good with quality q or less, and $T(i, r, \lambda)$ is the associated payment, subject to:

- **Feasibility:** $\mathbb{E}_{(i,r,\lambda)}\left[\Gamma(q|i, r, \lambda)\right] \geq F(q), \forall q \in [0, 1]$;

- **IC constraint:** Each agent (i, r, λ) reports (r, λ) truthfully;

- **IR constraint:** $U(i, r, \lambda) \equiv r \int q \, d\Gamma(q|i, r, \lambda) - T(i, r, \lambda) \geq 0$;

- **Non-negative transfers:** $T(i, r, \lambda) \geq 0, \forall (i, r, \lambda)$.

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The designer maximizes, for some constant $\alpha \geq 0$, a **weighted sum of revenue and agents' utilities**:

$$\mathbb{E}_{(i,r,\lambda)} [\alpha T(i, r, \lambda) + \lambda U(i, r, \lambda)].$$

Comments about the model

Lemma: It is without loss of optimality for the mechanism designer to only elicit information about r through the mechanism—allocation and transfers depend on (i, r) but not on λ directly.

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This implies that the designer forms expectations over the unobserved social welfare weights:

$$\lambda_i(r) \equiv \mathbb{E}_{(i,r,\lambda)} \left[\lambda \mid i, r \right];$$

we call these functions **Pareto weights**, and assume they are continuous.

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Designer's objective becomes:

$$\mathbb{E}_{(i,r)} \left[\alpha T_i(r) + \lambda_i(r) U_i(r) \right].$$

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Economic idea:
The designer assesses the "need" of agents by estimating the unobserved welfare weights based on the observable (label i) and elicitable (wtp r) information.

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Economic idea:
The designer assesses the "need" of agents by estimating the unobserved welfare weights based on the observable (label i) and elicitable (wtp r) information.

Consequence:
The optimal allocation depends on the **statistical correlation** of labels and willingness to pay with the unobserved social welfare weights.

Examples:

- If label i captures income brackets, then we may naturally think that

$$\mathbb{E}[\lambda \,|\, i] \equiv \bar{\lambda}_i \geq \bar{\lambda}_j \equiv \mathbb{E}[\lambda \,|\, j]$$

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- Suppose there is just one label $I = \{i\}$, but we elicit information about willingness to pay of two patients for a cancer treatment:

  Agent A: $50,000

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- Other possibilities: Labels could capture family status, ethnicity, race, educational background, neighborhood etc.

- **Many labels,** i **captures low income**, and we elicit information about willingness to pay of two patients for a cancer treatment:

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- Suppose there is just one label $I = \{i\}$, but we elicit information about willingness to pay of two **consumers for a concert ticket**:

  Agent A: \$50

  Agent B: \$0

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The role of revenue:

- The weight α on revenue captures the value of a dollar in the designer's budget, spent on the most valuable "cause."

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- When $\alpha > \bar{\lambda}_i$ for all i, there is an "**outside cause**".

- When $\alpha < \bar{\lambda}_i$ for some i, **lump-sum payments to agents in group i are prohibited or costly**.

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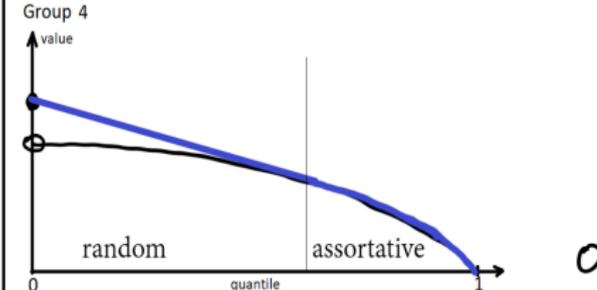Interpretation: A vast majority of agents have a willingness to pay that is bounded away from zero.

Examples: housing, food, basic health care

Derivation of Optimal Mechanism

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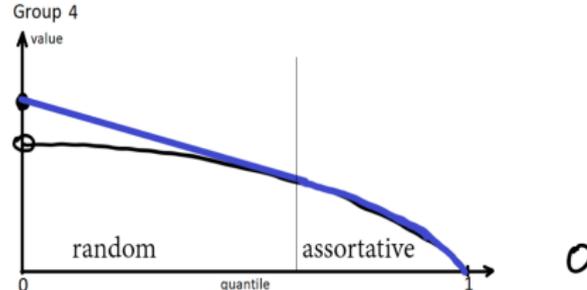2. The objects are allocated "within" groups: For each label i, the objects $F_i^\star$ are allocated optimally to agents in group i.

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Observation: Only expected quality, $Q_i(r)$, matters for payoffs.

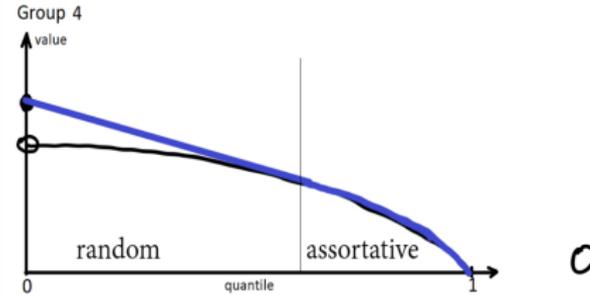# Derivation of Optimal Mechanism

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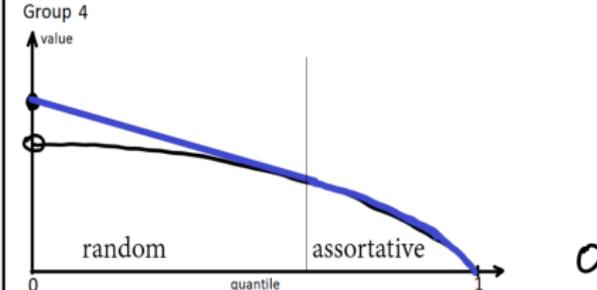$$Q_i(r) = (F_i^\star)^{-1}(G_i(r)), \ \forall r \in [a, b];$$

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- **Random matching (non-market allocation)**:
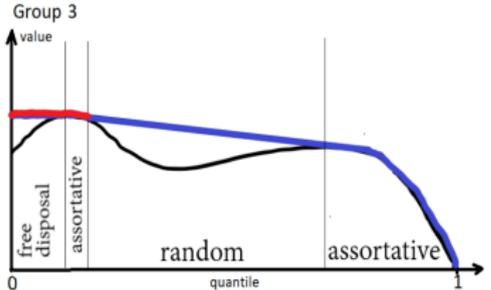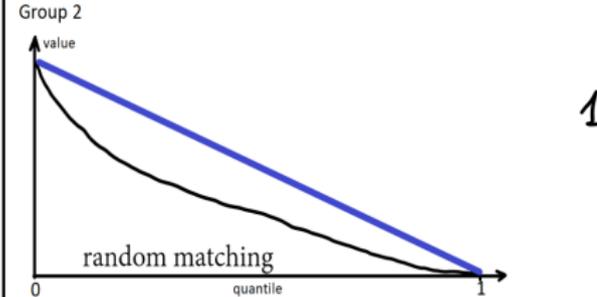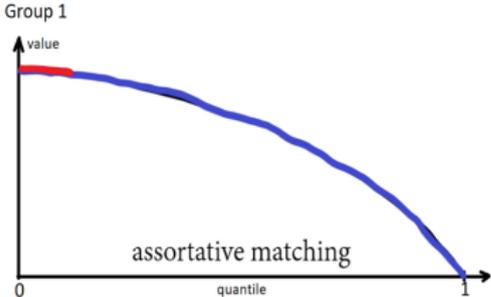
$$Q_i(r) = \bar{q}, \, \forall r \in [a, b].$$

Derivation of Optimal Mechanism: within-group allocation

Group 1

value

0   quantile   1

Group 2

value

0   quantile   1

Group 3

value

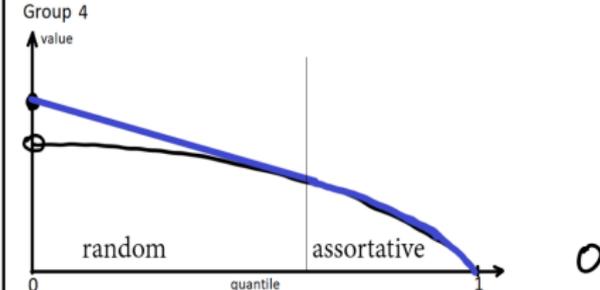0   quantile   1

Group 4

value

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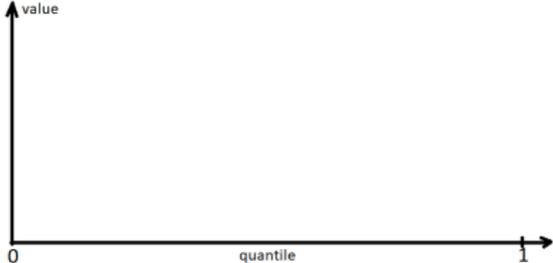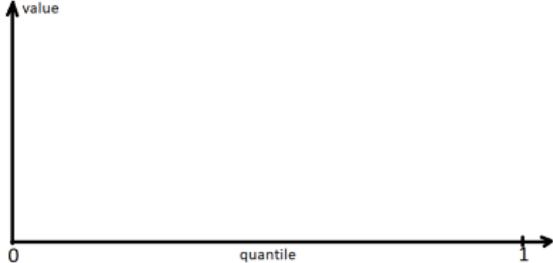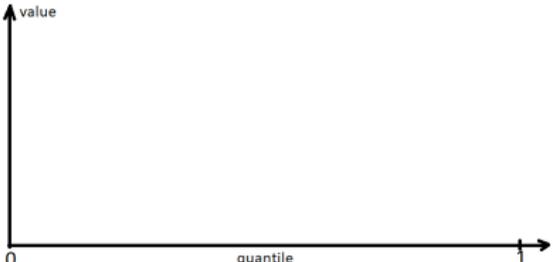
Derivation of Optimal Mechanism: within-group allocation
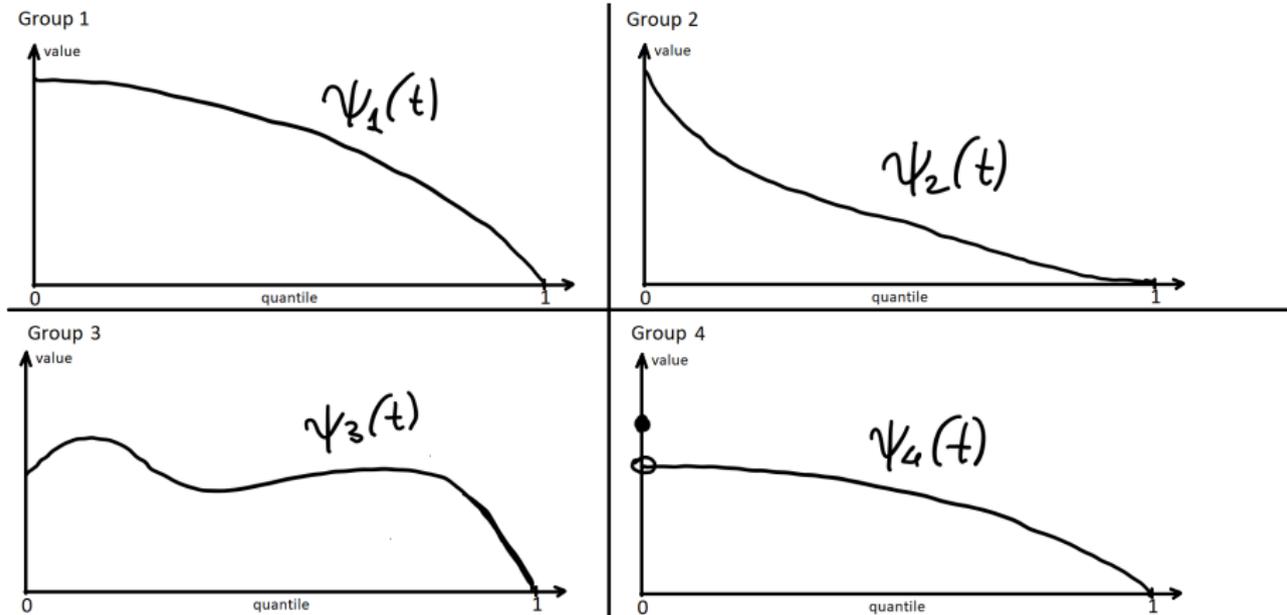
$$\Psi_i(t) := \int_t^1 V_i(G_i^{-1}(x))dx + \max\{0, \bar{\lambda}_i - \alpha\}\, \underline{r}_i\, \mathbf{1}_{\{t=0\}}.$$

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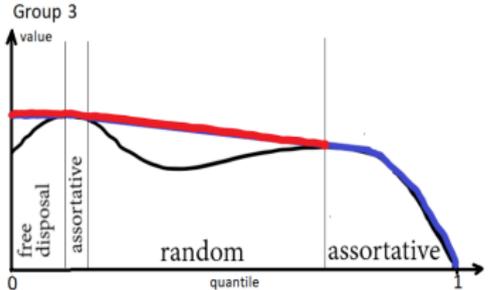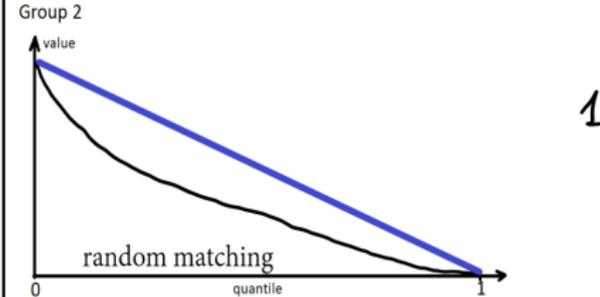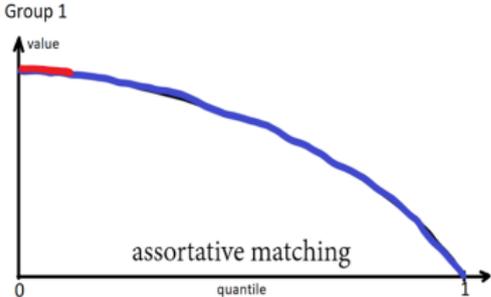
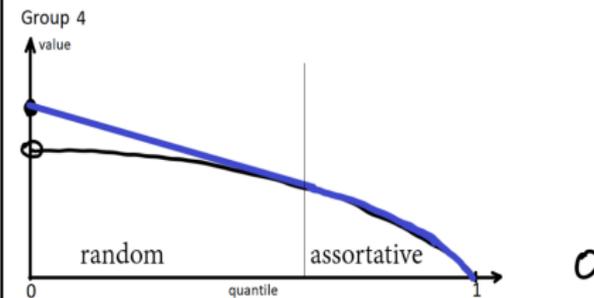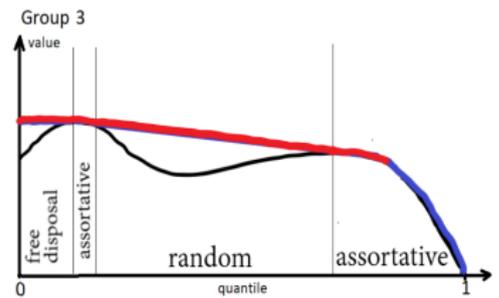# Derivation of Optimal Mechanism: within-group allocation

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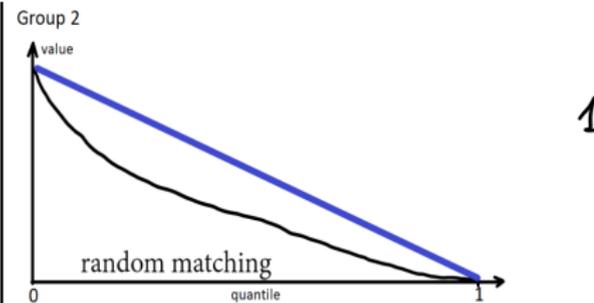# Derivation of Optimal Mechanism: across-group allocation

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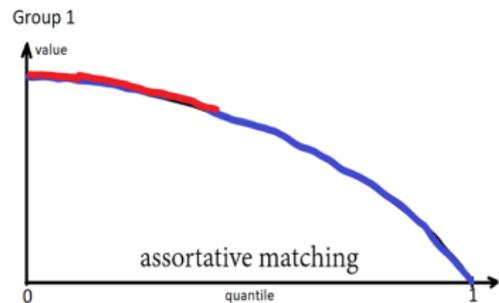
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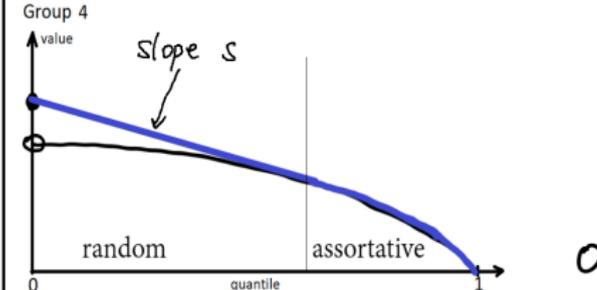# Derivation of Optimal Mechanism: across-group allocation

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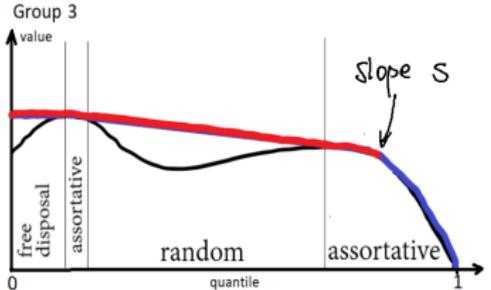# Derivation of Optimal Mechanism: across-group allocation

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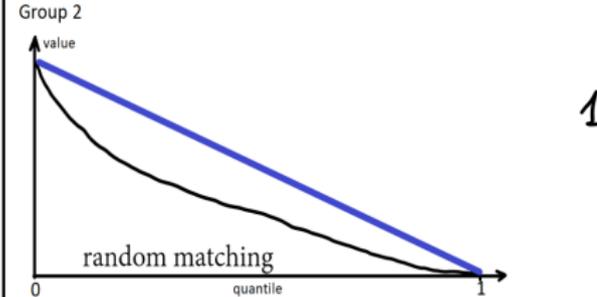# Derivation of Optimal Mechanism: across-group allocation

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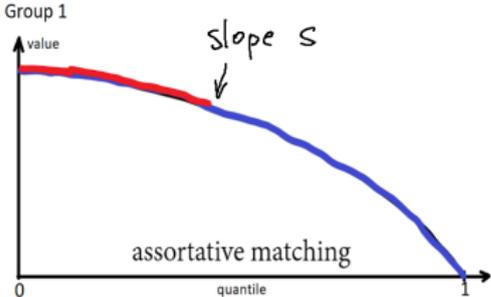# Derivation of Optimal Mechanism: across-group allocation

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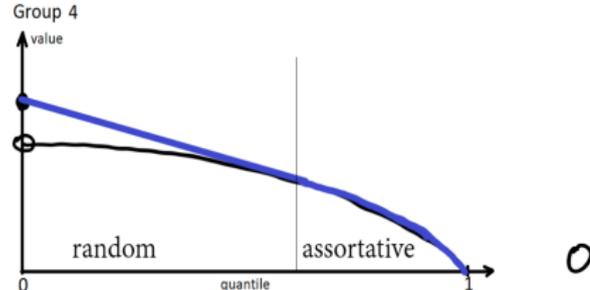# Derivation of Optimal Mechanism: across-group allocation

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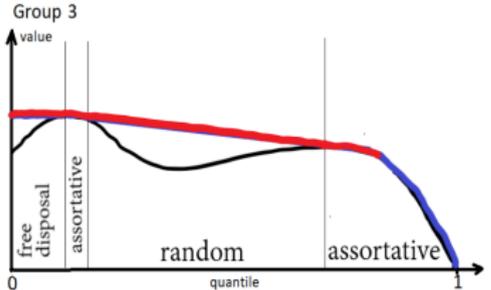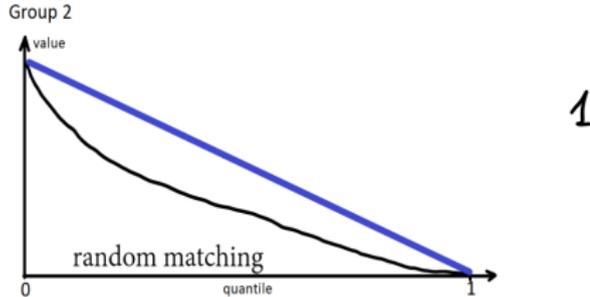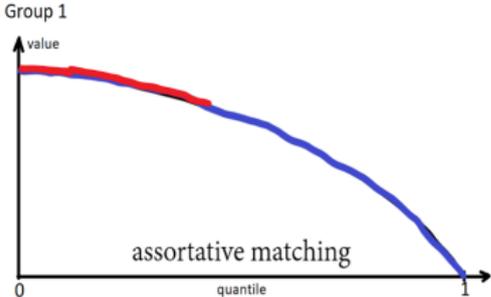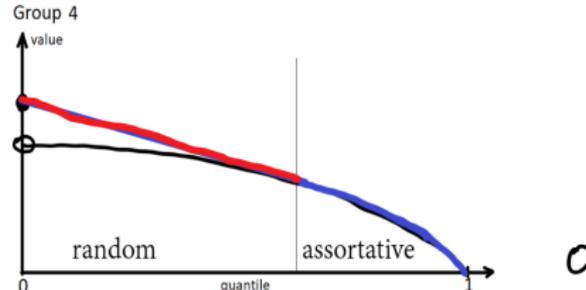# Derivation of Optimal Mechanism: across-group allocation

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Economic Implications

Proposition 1 (Affirmative action and in-kind transfers)

If the average Pareto weight $\bar{\lambda}_i$ for group i is strictly larger than the weight on revenue α, and the good is essential ($\underline{r}_i > 0$), then there exists $r_i^\star > \underline{r}_i$ such that the allocation is random at a price of 0 for all types $r \leq r_i^\star$.

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"Wrong" intuition: A random allocation for free increases the welfare of agents with lowest willingness to pay

Correct Intuition: A random allocation for free enables the designer to lower prices for all agents

▸ Picture

Proposition 2 (A fully random allocation)

A necessary condition for full randomization to be optimal within group i is that

$$\alpha \bar{r}_i \leq \int_{\underline{r}_i}^{\bar{r}_i} r \lambda_i(r) dG_i(r). \tag{1}$$

A sufficient condition is (1) and quasi-convexity of $\alpha J_i(r) + \Lambda_i(r) h_i(r)$, where $\Lambda_i(r) = \mathbb{E}^i[\lambda_i(\tilde{r}) | \tilde{r} \geq r]$, and h_i is the inverse hazard rate of G_i.

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A necessary condition for full randomization to be optimal is that lump-sum transfers are **not** feasible ($\alpha < \bar{\lambda}_i$).

Proposition 3 (Assortative matching at the top)

If Pareto weights are non-increasing, and $\alpha \geq \bar{\lambda}_i$, then there is assortative matching at the top, that is, for all types $r \geq r_i^\star$ for some $r_i^\star > \underline{r}_i$.

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The assumptions guarantee that the revenue-maximizing motive dominates for high enough types.

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The assumptions guarantee that the revenue-maximizing motive dominates for high enough types.

The assumption $\alpha \geq \bar{\lambda}_i$ holds when lump-sum transfers are allowed.

Even if there is random allocation "at the bottom," it might be a good idea to have a price gradient "at the top."

Proposition 4 (Fully assortative matching)

Suppose that $\alpha \geq \bar{\lambda}_i$ and $\underline{r}_i = 0$. Assortative matching is optimal if and only if $\alpha J_i(r) + \Lambda_i(r) h_i(r)$ is non-decreasing, where

$$h_i(r) = \frac{1 - G_i(r)}{g_i(r)}, \ \ J_i(r) = r - h_i(r), \ \ \Lambda_i(r) = \mathbb{E}^i[\lambda_i(\tilde{r}) \mid \tilde{r} \geq r].$$

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Assortative matching is suboptimal if **willingness to pay is strongly (negatively) correlated with the unobserved social welfare weights**.

Proposition 5 (Intermediate quality to the random-allocation group)

Suppose that there are two groups and that in group 1 there is assortative matching (of allocated objects), and in group 2 there is fully random matching.

Then, there exist $\underline{q} \leq \bar{q}$ such that group 2 gets objects of quality $[\underline{q}, \bar{q}]$, and group 1 gets objects of quality $[0, \underline{q}) \cup (\bar{q}, 1]$.

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Intuition: Under random matching, only expected quality matters.

But under assortative matching, it is key to have dispersion in quality (easiest to see for revenue maximization—the allocation of lower types is reduced to minimize the information rents of higher types).

Conclusions and Market-design Implications

Market-design Implications

When to use in-kind redistribution? (random allocation of a range of quality levels at a below-market price)

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   - Health care for general population (e.g. in some European countries);

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2. **Efficiency maximization**: When the welfare weights are not strongly correlated with willingness to pay:

   ▸ Small dispersion in welfare weights to begin with;

   ▸ Large dispersion in welfare weights but little correlation with WTP (affordable goods for which WTP depends heavily on taste).

Conclusions

- We proposed a model of **optimal object allocation under general redistributive concerns**.

- Our analysis uncovers the importance of three factors:

  1. Correlation between the unobserved welfare weights and the information that the designer can elicit or observe directly;

  2. The role of raising revenue and availability of lump-sum transfers;

  3. Whether the good is essential or not.

- More work is needed on **mechanism design with redistributive concerns**:

  1. Relaxing linearity assumptions (income effects, risk aversion);

  2. Considering ex-post fairness;

  3. Interaction with conventional redistributive tools.

Appendix

Additional results

Proposition 6 (Non-monotonicity in the use of randomization)

Consider a sequence of non-increasing Pareto weights $\lambda_i^n(r)$ such that $\alpha \geq \bar{\lambda}_i^n$, and assume that $J_i'(r) \geq \underline{J}_i > 0$. If either

- *for all r and $\epsilon > 0$, $|\lambda_i^n(r) - \bar{\lambda}_i^n| < \epsilon$ for large enough n, or*
- *for all $r > \underline{r}$ and $\epsilon > 0$, $\lambda_i^n(r) < \epsilon$ for large enough n,*

then any optimal solution converges point-wise to assortative matching (of objects that are allocated).

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There is non-monotonicity in how the correlation between WTP and welfare weights influences the use of non-market mechanisms.

Intuition: When weights become extremely skewed towards lowest-WTP agents but the average stays bounded, the weights on all higher-WTP agents are converging to 0.

Proposition 7 (Across-group allocation with random matching)

Suppose that under the optimal mechanism, all groups have fully random matching. Relabel the groups so that lower $i = 1, ..., |I|$ corresponds to lower

$$\int_{\underline{r}_i}^{\bar{r}_i} \tau \lambda_i(\tau) dG_i(\tau).$$

Then, there exists an optimal mechanism in which

$$supp(F_i^\star) = [q_i, q_{i+1}] \cap supp(F),$$

where $q_1 = \min supp(F)$ and $q_{|I|+1} = \max supp(F)$.

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In this case, the qualities allocated to different groups are effectively disjoint.

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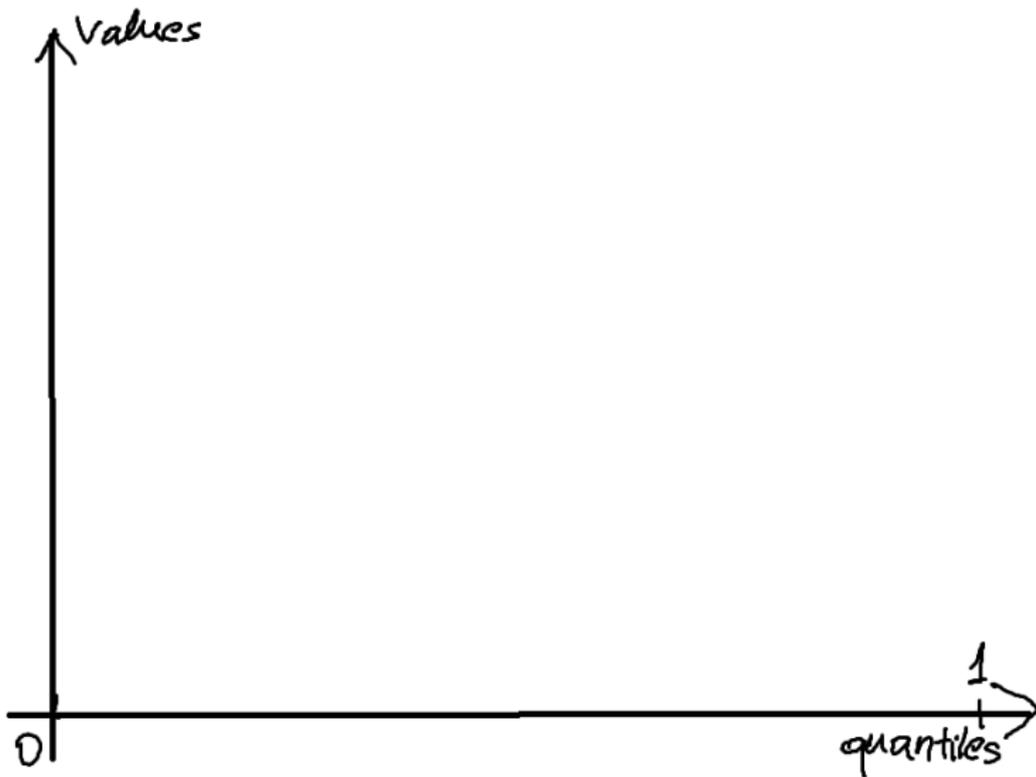
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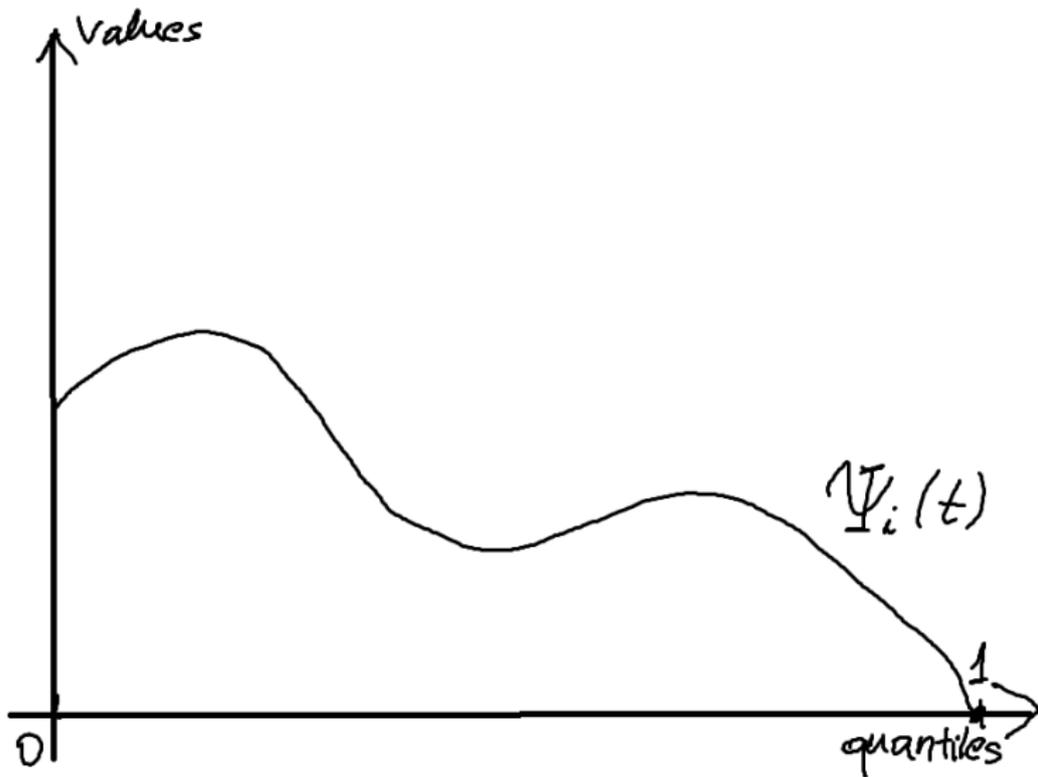*where $\bar{q}_i \leq \bar{q}_{i+1}$.*

Moreover, if $\underline{r}_i = 0$ and $\alpha > \bar{\lambda}_i$ for all i, then all $\underline{q}_i = \min supp(F)$.

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Moreover, if $\underline{r}_i = 0$ and $\alpha > \bar{\lambda}_i$ for all i, then all $\underline{q}_i = \min supp(F)$.

In this case, the qualities allocated to different groups overlap; under conditions, all groups have the same lower bound on quality; the upper bound depends only on the highest WTP within a group.

$\Psi_i(t)$

$co(\Psi_i)(t)$

$\Psi_i(t)$

Values

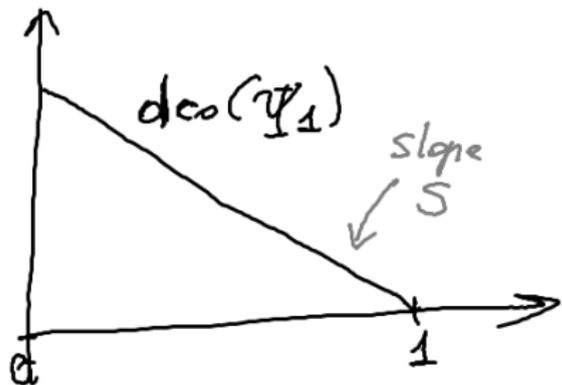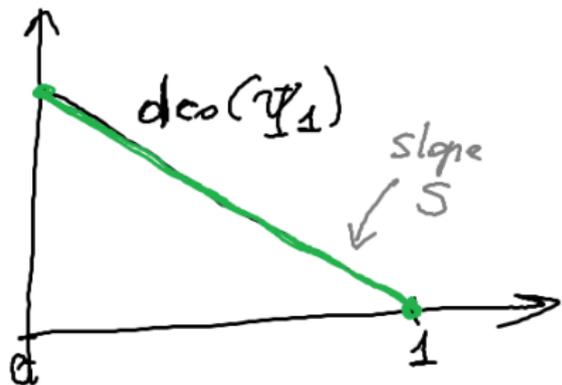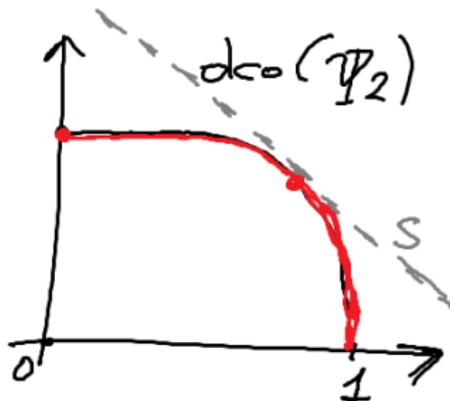1

0

quantiles

Values

$dco(\Psi_i)(t)$

$co(\Psi_i)(t)$

$\Psi_i(t)$

1

0
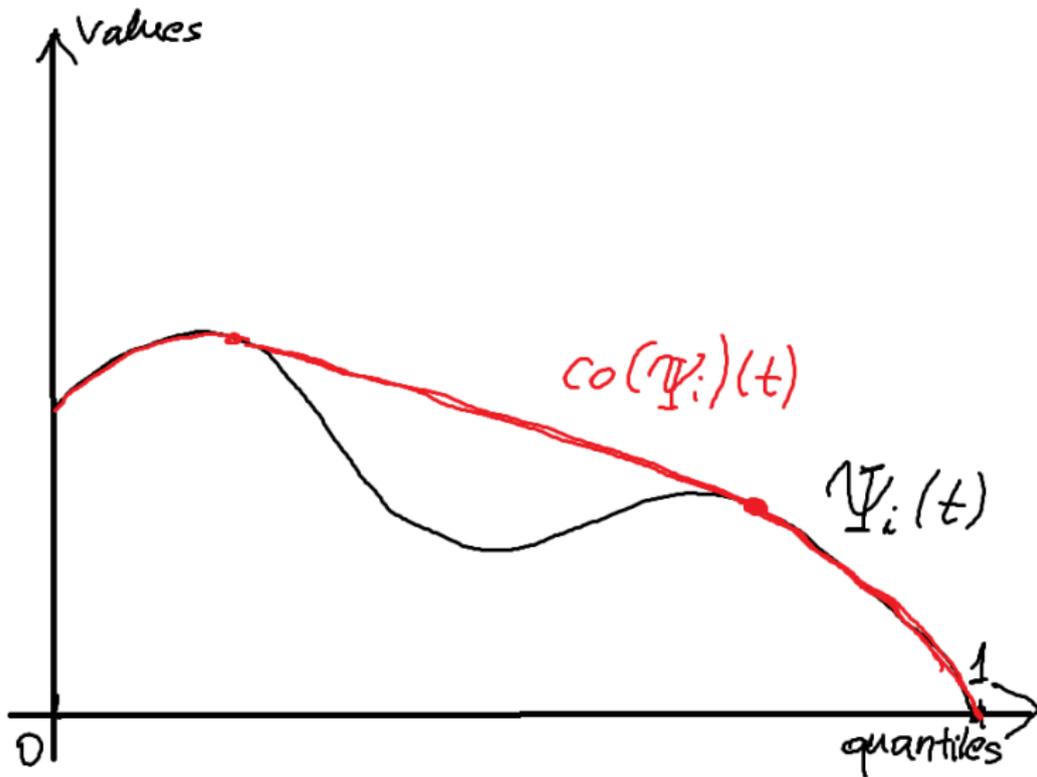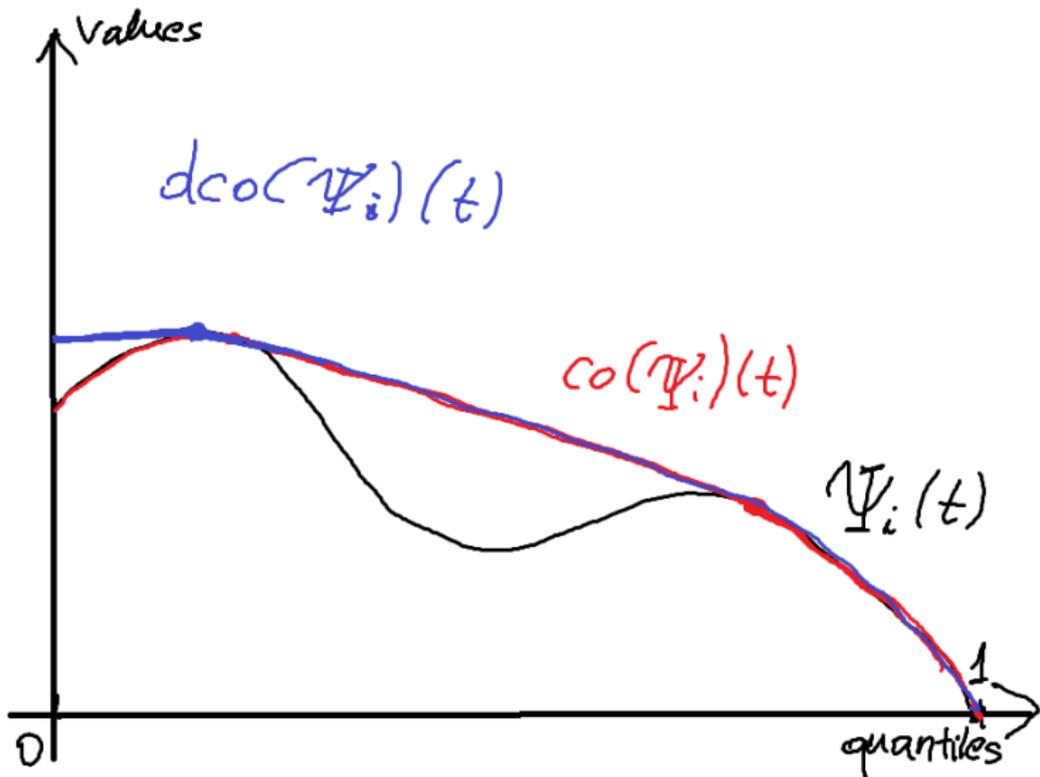
quantiles
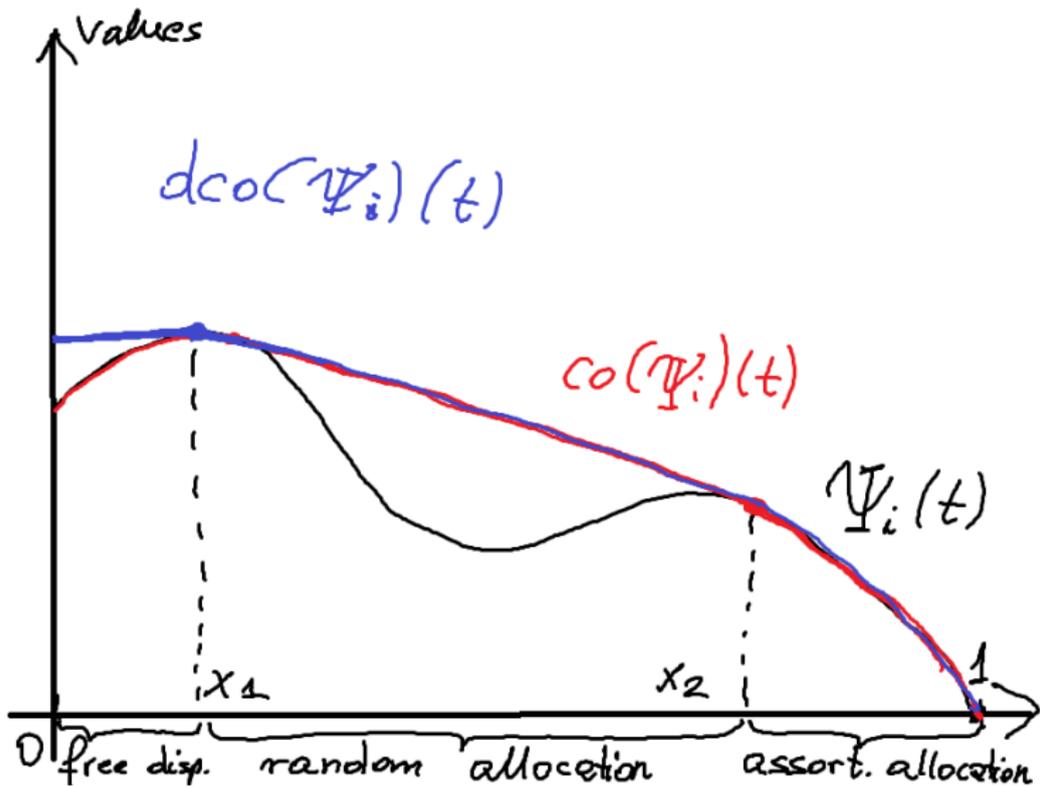
Back to slides

Group 1

Group 2

$dco(\Psi_1)$

$dco(\Psi_2)$

a

1

0

1

0

1

quality

Group 1

$dco(\Psi_1)$

slope
S

Group 2

$dco(\Psi_2)$

S

0

1

group 2

1

quality

Group 1

$dco(\psi_1)$

slope
S

Group 2

$dco(\psi_2)$

S

O

group 2   group 1
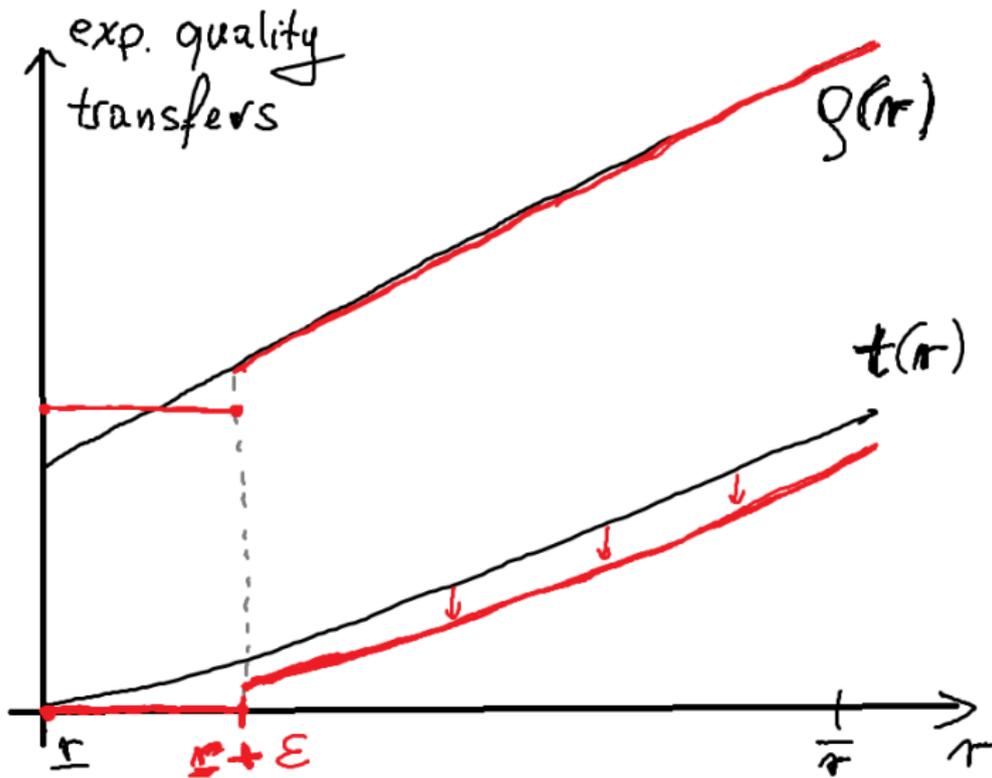
1

quality

Group 1

Group 2

dco(Ψ_1)

dco(Ψ_2)

slope S

S

0

1

group 2

group 1

group 2 quality

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exp. quality
transfers

$g(r)$

$t(r)$

$\underline{r}$ $\underline{r} + \varepsilon$ $\bar{r}$ r

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exp. quality transfers

$$U(r) = U(\underline{r}) + \int_{\underline{r}}^{r} \varrho(\tau)\,d\tau$$

$\varrho(r)$

first-order gain

$\underline{r}$ $\underline{r} + \varepsilon$ $\overline{r}$ r

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exp. quality transfers

$g(r)$

at most a second-order loss

first-order gain

ε

$\underline{r}$

$\underline{r} + \varepsilon$

$\overline{r}$

r