

# Redistribution through Markets

Scott Duke Kominers

Harvard Business School and Department of Economics, Harvard University

(joint work with Piotr Dworzak<sup>®</sup> Mohammad Akbarpour<sup>1</sup>)

Virtual Market Design Seminar

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<sup>1</sup>Authors' names are in (ce<sup>®</sup>tified) random order.

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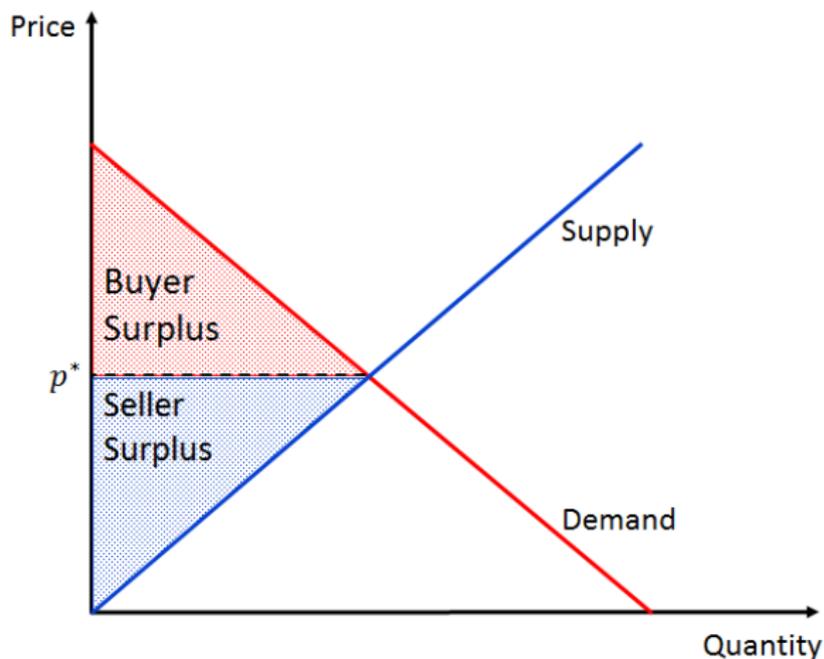
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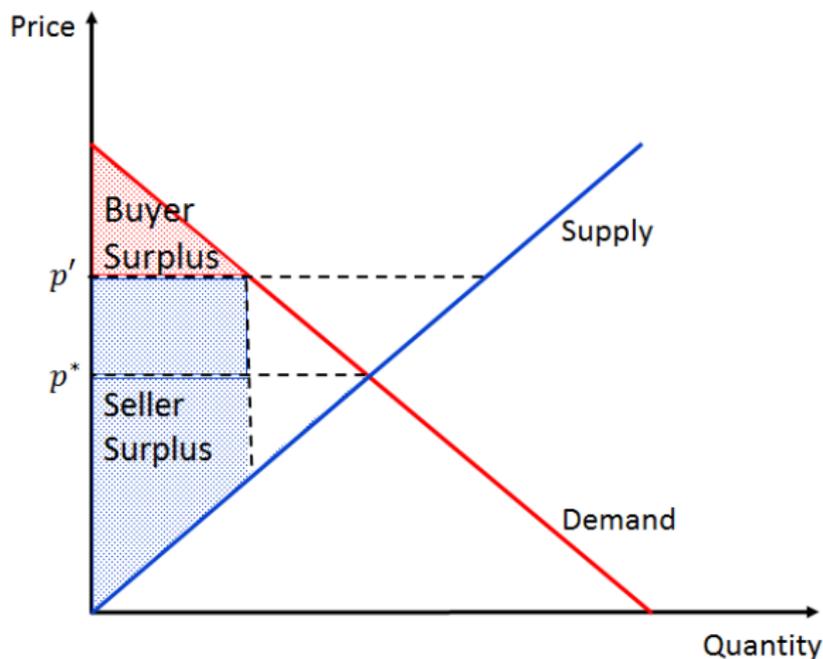
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- Knee-jerk economics answer: NO!

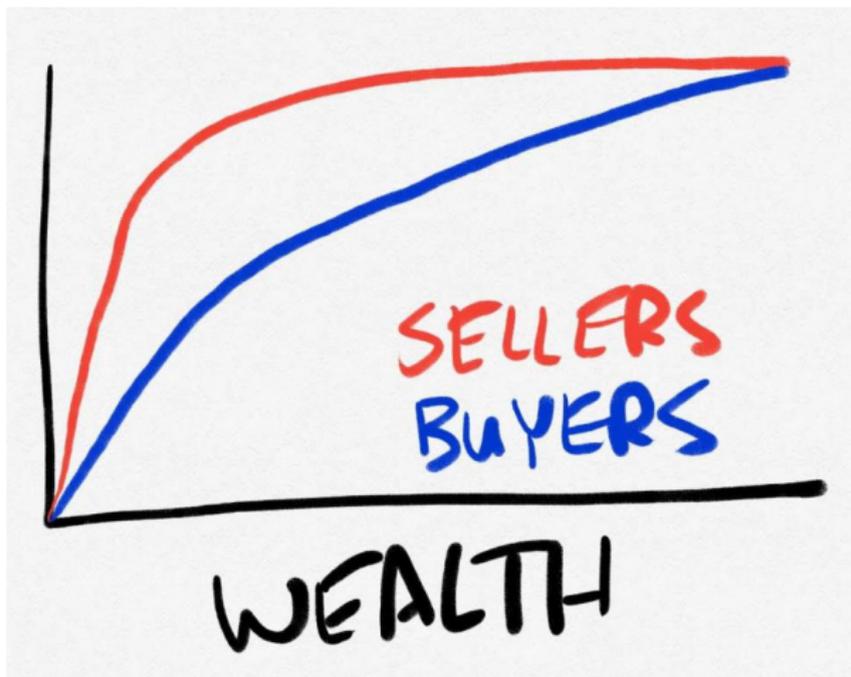
# Standard Economic Intuition



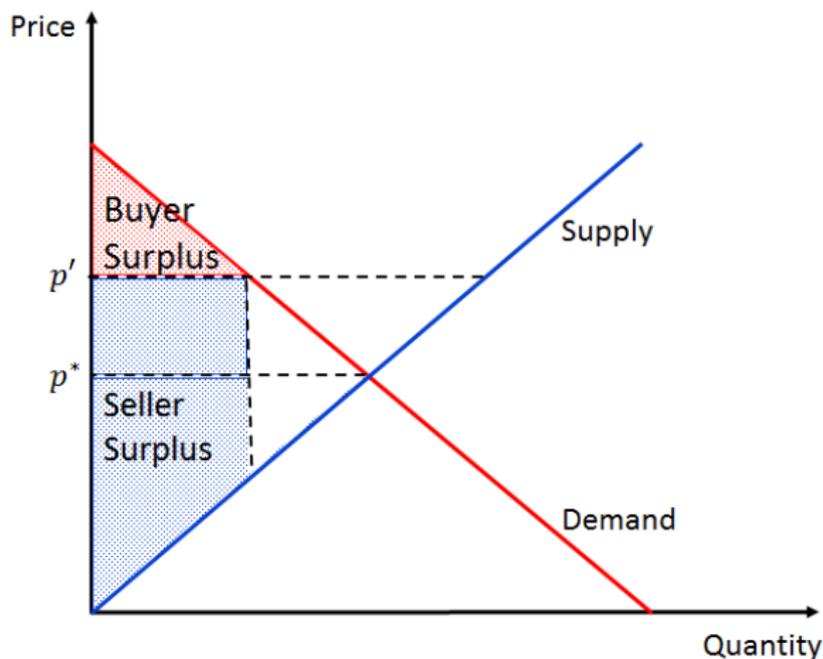
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# Buyer/Seller Inequality



# Standard Economic Intuition – revisited



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  - As sellers become (systematically) poorer than buyers, the competitive-equilibrium price decreases.
  - However, when sellers are poorer, the designer would like them to have *more* money on the margin, all else equal.
- ⇒ Past some point, competitive-equilibrium pricing will not be socially optimal(!).

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- ★ Model inequality as dispersion in marginal values for money.<sup>1,2</sup>
  - ↔ allowing for arbitrary Pareto weights in mechanism design.
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  - same-side inequality  $\Rightarrow$  rationing

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## Market Design under Wealth Inequality

- We show how the form of the optimal mechanism depends on the type of inequality: **cross-side** vs. **same-side**.

## Conceptual Contribution

- Competitive equilibrium is not necessarily “optimal” in the presence of wealth inequality.

# Related Literature (not exhaustive!)

**Rationing vs. Market Mechanism:** Weitzman (1977); Condorelli (2013); Huesmann (2017)

**Auctions with Budget-Constrained Bidders:** Che–Gale (1998); Fernandez–Gali (1999); Che–Gale–Kim (2012); Pai–Vohra (2014); Kotowski (2017)

**Optimal Tax:** Diamond–Mirrlees (1971); Atkinson–Stiglitz (1976); Piketty–Saez (2013); Scheuer (2014), Saez–Stantcheva (2016, 2017); Scheuer–Werning (2017)

**Minimum Wage:** Allen (1987); Guesnerie–Roberts (1987); Boadway–Cuff (2001); Lee–Saez (2012); Cahuc–Laroque (2014)

**Fair Marketplace Design:** Hylland–Zeckhauser (1979); Bogomolnaia–Moulin (2001); Budish (2011)

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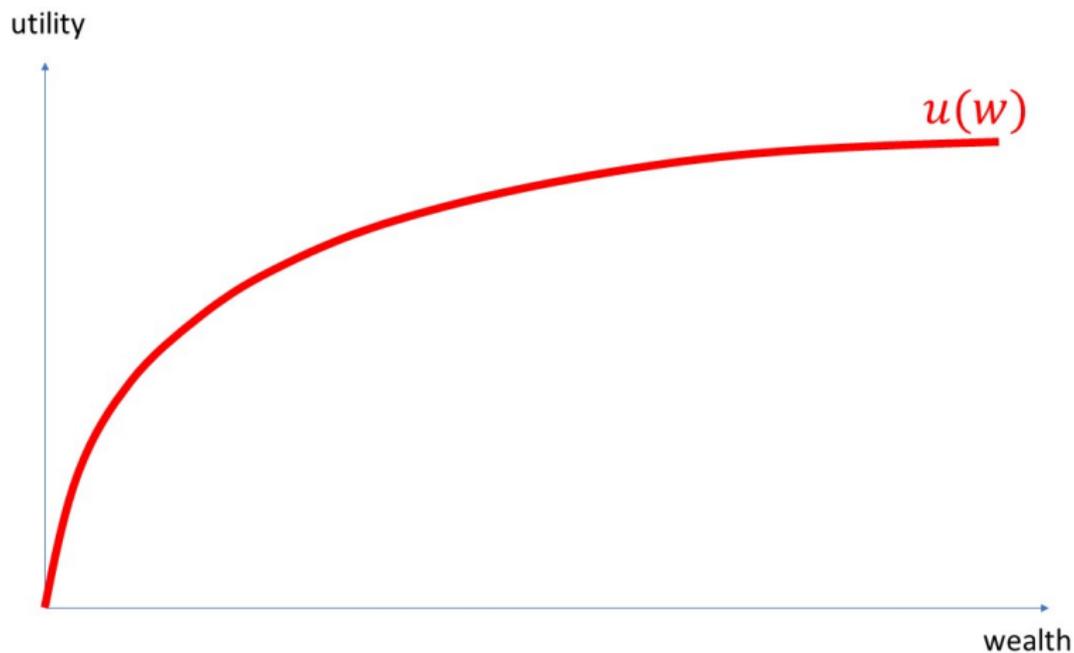
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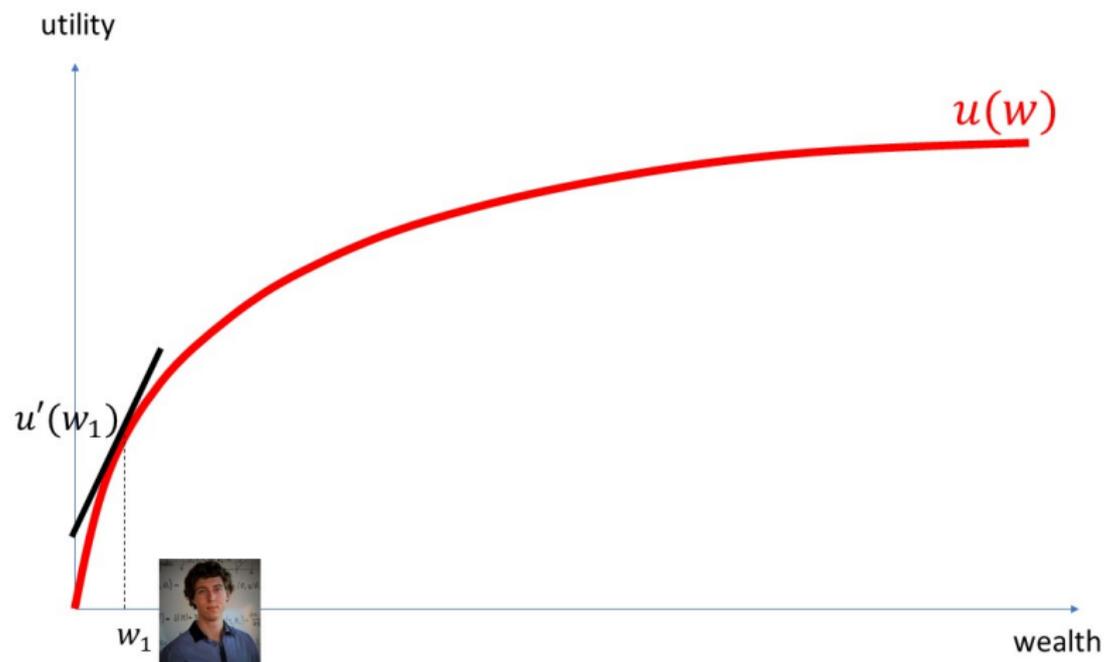
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→ How to maximize welfare? Set price  $p^{\text{CE}}$ .

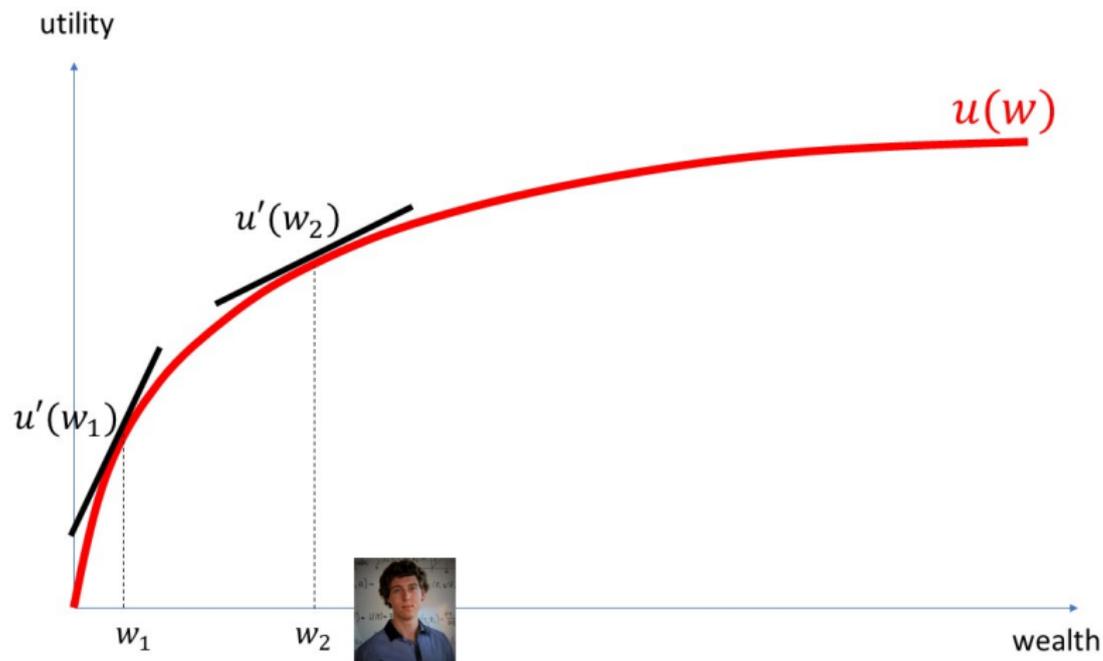
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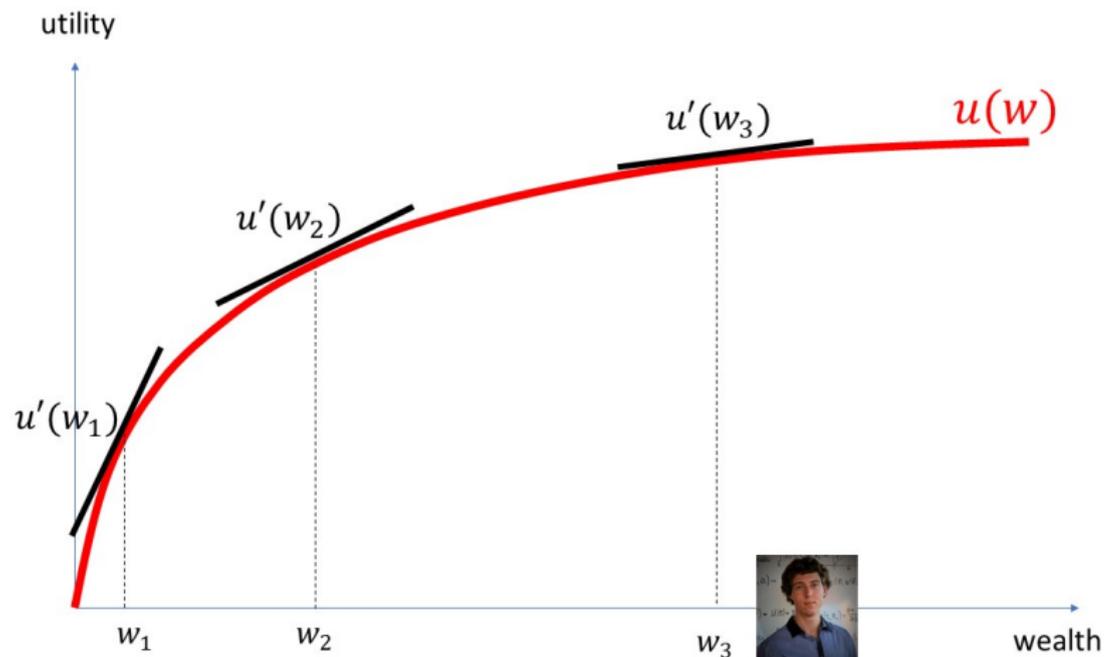
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↪ Rates of substitution  $\frac{v^K}{v^M} =: r \sim G_j(r)$  ( $\sim \text{Unif}[0, 1]$  for now).

# Simple Mechanisms under Uniform Distribution

- First, we solve one-sided problems given  $Q$  and  $R$ .
- Then, we link our characterizations of seller- and buyer-side solutions through the optimal choice of  $Q$  and  $R$ .
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## Key Observation

*Agents' behavior depends only on  $r = \frac{v^K}{v^M}$ , but  $r$  is informative about welfare weight  $\lambda(r) := \mathbb{E}[v^M \mid \frac{v^K}{v^M} = r]$ .*

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*⇒ Identifying “poorer” agents through market behavior.*

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- **same-side inequality on side**  $j \in \{B, S\}$  if  $\lambda_j \neq \Lambda_j$ 
  - **low** if  $\lambda_j(r_j) \leq 2\Lambda_j$ ; **high** otherwise

# The Optimal Seller Price $p_S$

GOAL: Acquire  $Q$  objects while spending at most  $R$ .

$$\max_{p_S \geq G_S^{-1}(Q)} \left\{ \frac{Q}{G_S(p_S)} \int_{r_S}^{p_S} \lambda_S(r)(p_S - r) dG_S(r) + \Lambda_S(R - p_S Q) \right\}.$$

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Three effects of pushing  $p_S$  above  $p_S^C$ :

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## Proposition

*When seller same-side inequality is low,  $p_S = p_S^C$  is optimal. When seller same-side inequality is high and  $Q$  is low enough, rationing at a price  $p_S > p_S^C$  is optimal.*

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- Decision to trade always identifies sellers with low rates of substitution ( $\Rightarrow$  equity  $\uparrow$ )!

# The Optimal Buyer Price $p_B$

GOAL: Allocate  $Q$  objects with revenue at least  $R$ .

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# Linking the Two Sides (II) – price wedges

## Proposition

*When same-side inequality is low on both sides, it is optimal to set prices such that the market clears,  $G_S(p_S) = \mu(1 - G_B(p_B))$ ; we redistribute any revenue as a lump-sum payment to the side with higher average value for money  $\Lambda_j$ .*

# Linking the Two Sides (III) – rationing

## Proposition

*If seller same-side inequality is high and  $\Lambda_S \geq \Lambda_B$ , then (if  $\mu$  is low enough) it is optimal to ration the sellers by setting a single price above the market-clearing level.*

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*If buyer same-side inequality is high and buyers' willingness to pay is sufficiently high [relative to the seller's rates of substitution], then it is optimal to ration the buyers for  $\mu \in (1, 1 + \epsilon)$ .*

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- *Total number of prices involved is small (4 at most; 3 if nonzero lump-sum transfer).*

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- 4 The solution is characterized as a concave closure of the objective function, subject to a linear constraint.
- 5 By Carathéodory's Theorem, the solution is supported on at most three points  $\rightsquigarrow$  three-price characterization for each side. (And two constraints are common across the market!)

# When to use each instrument? (modulo reg. conditions)

- Cross-side inequality ( $\Lambda_B \neq \Lambda_S$ )  $\rightsquigarrow$  price wedge.
  - $\rightsquigarrow$  moves money to the poorer side of the market.
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  - $\leadsto$  shifts more money to the sellers who are most eager to trade – identifies the “poorest.”
- High buyer-side inequality + large amount of trade  $\leadsto$  rationing with multiple prices.
  - N.B. The “poorest” buyers are those that are *least able* to trade!
  - $\leadsto$  Rationing at a single price helps richer buyers more than poorer.

# Optimal Market Design under Wealth Inequality

SAME-SIDE  
INEQUALITY

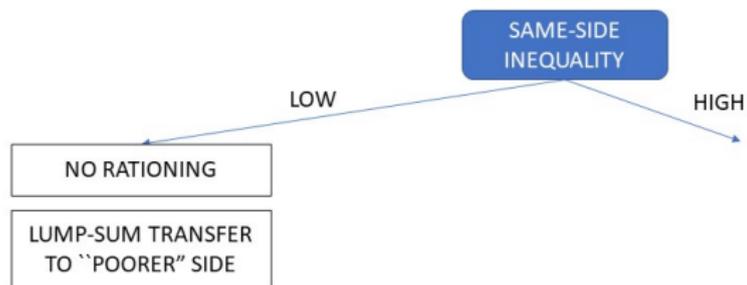
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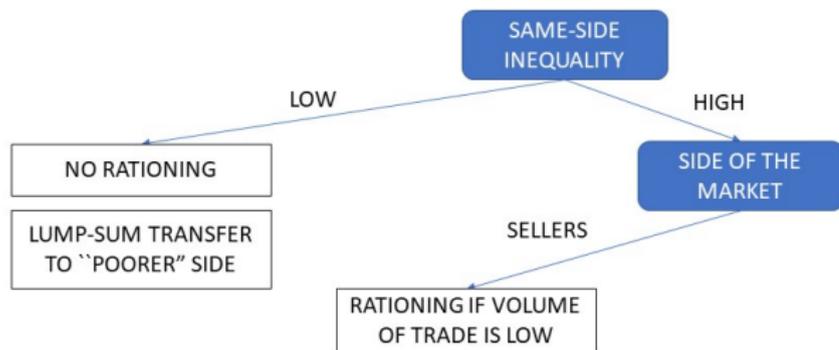
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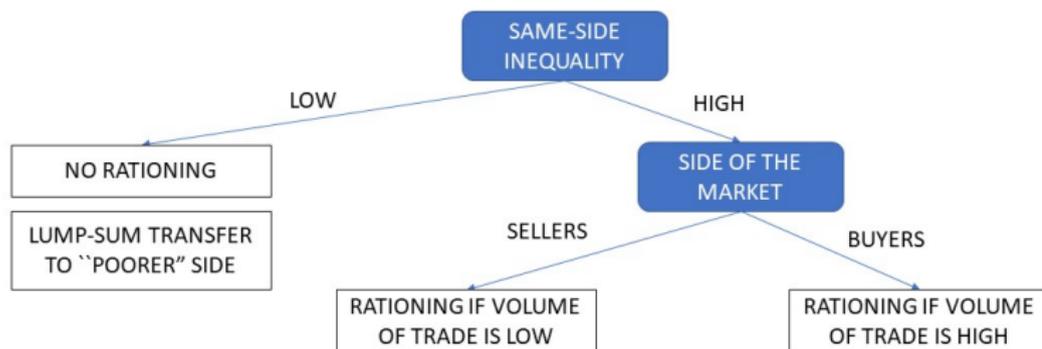
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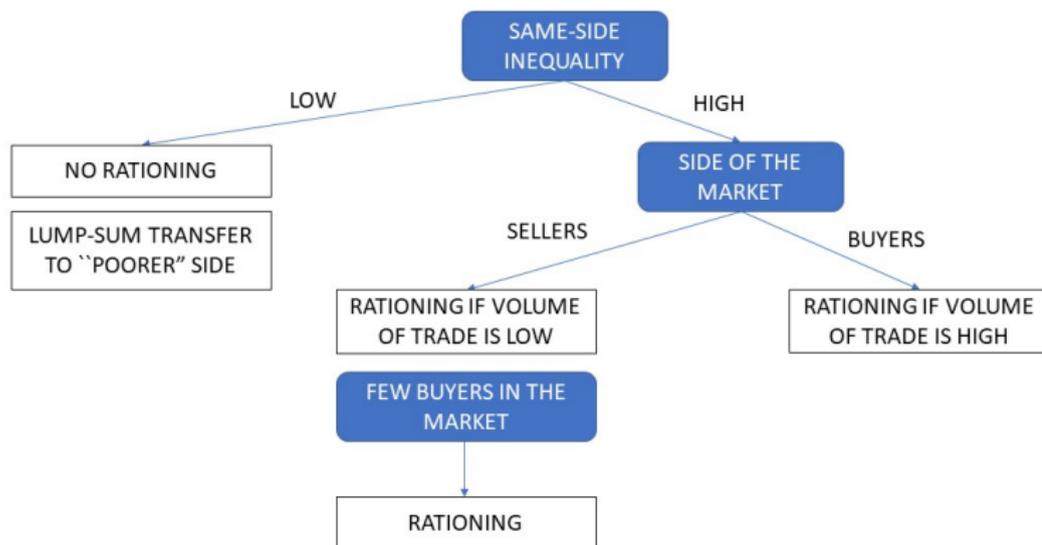
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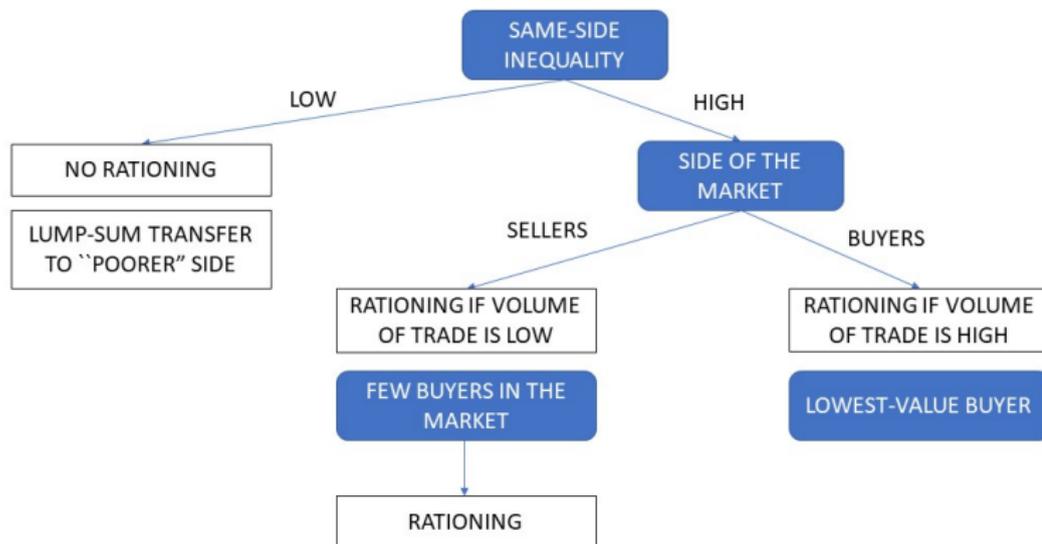
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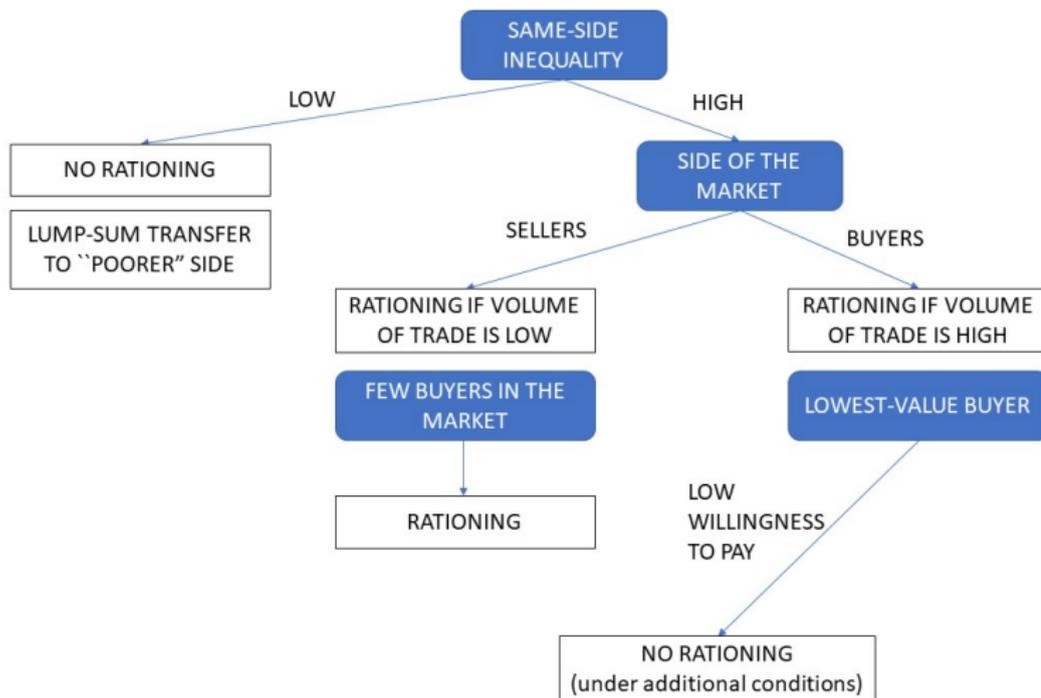
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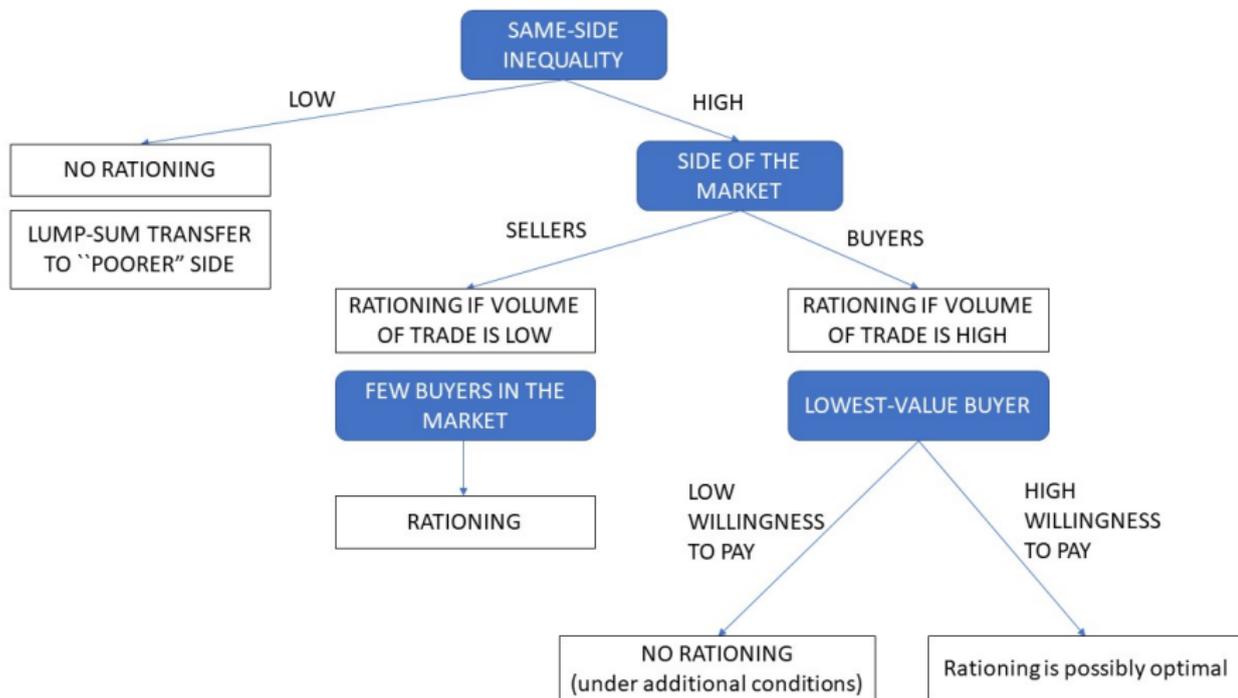
# Optimal Market Design under Wealth Inequality



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# Optimal Market Design under Wealth Inequality



# Coming Soon: New Paper!

## Redistributive Allocation Mechanisms

Mohammad Akbarpour <sup>®</sup> Piotr Dworzak <sup>®</sup> Scott Duke Kominers\*

April 14, 2020 – PRELIMINARY (AND SOMEWHAT INCOMPLETE)

### Abstract

Many scarce public resources are allocated below market-clearing prices (and sometimes for free). Such “non-market” mechanisms necessarily sacrifice some surplus, yet they can potentially improve equity by increasing the rents enjoyed by agents with low willingness to pay. In this paper, we develop a model of mechanism design with redistributive concerns. Agents are characterized by a privately observed willingness to pay for quality, and a publicly observed label. A market designer controls allocation and pricing of a set of objects of heterogeneous quality, and maximizes a linear combination of revenue and total surplus—with Pareto weights that depend both on observed and unobserved agent characteristics. We develop methods to solve for an optimal mechanism in this framework, and derive conditions for optimality of certain simple mechanisms such as allocating the objects assortatively at “market” prices, fully random allocation at a price of 0, and hybrid mechanisms that offer the lowest-quality objects for free using a lottery and allocate higher-quality objects at strictly positive prices.

**Keywords:** optimal mechanism design, redistribution, inequality, welfare

**JEL codes:** C78, D47, D61, D63, D82

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  - Why? Past some point, revenue is a better way to redistribute!

## Bloomberg Opinion

## Economics

# Keep Sanitizer Out of the Market's Invisible Hand

Price gouging represents a shift in essential goods from the poor to the wealthy at the worst time.

By [Scott Duke Kominers](#)

March 16, 2020, 2:56 PM EDT



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# Food-Stamp Work Requirements Just Look Cruel

The rule doesn't help beneficiaries find the steady employment that doesn't exist.

By [Scott Duke Kominers](#)

January 27, 2020, 7:30 AM EST



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# Wrap

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\end{talk}

# Main Argument (I)

$\rightsquigarrow$  Maximize over  $H_S, H_B \in \Delta([0, 1])$ ,  $\underline{U}_B, \underline{U}_S \geq 0$

$$\mu \int_0^1 \phi_B^\alpha(q) dH_B(q) + \int_0^1 \phi_S^\alpha(q) dH_S(q)$$

subject to

$$\mu \int_0^1 q dH_B(q) = \int_0^1 q dH_S(q).$$

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$\rightsquigarrow$  Maximize over  $H_S, H_B \in \Delta([0, 1])$ ,  $\underline{U}_B, \underline{U}_S \geq 0$

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where  $\Lambda_j = \int \lambda_j(r) dG_j(r)$  is the average weight of  $j$  and

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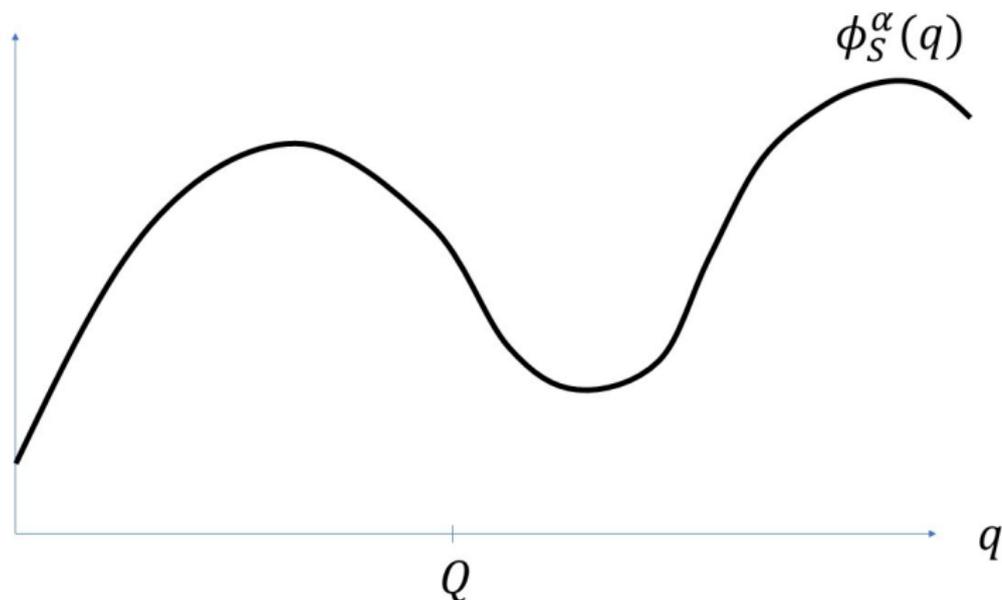
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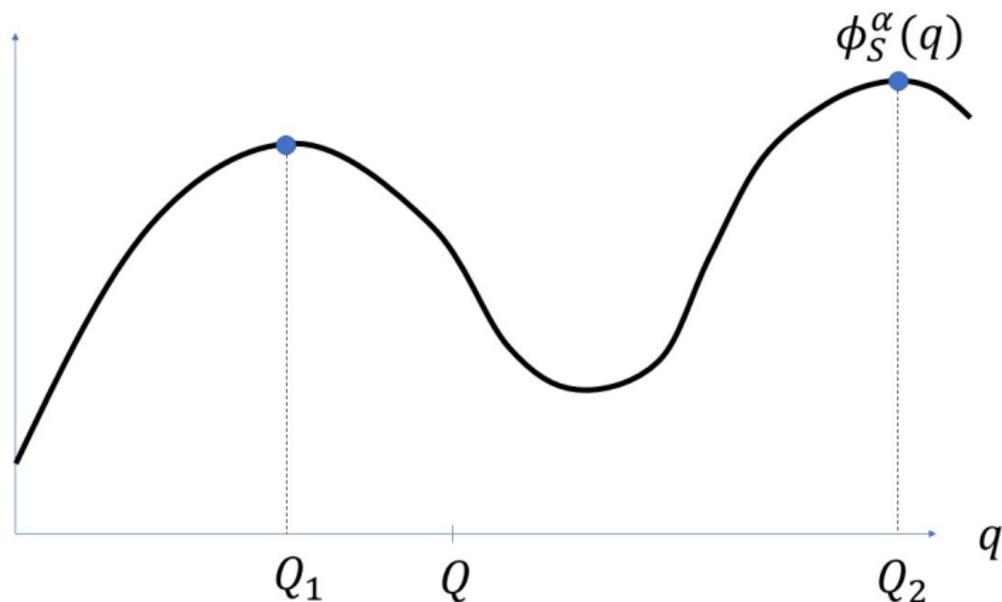
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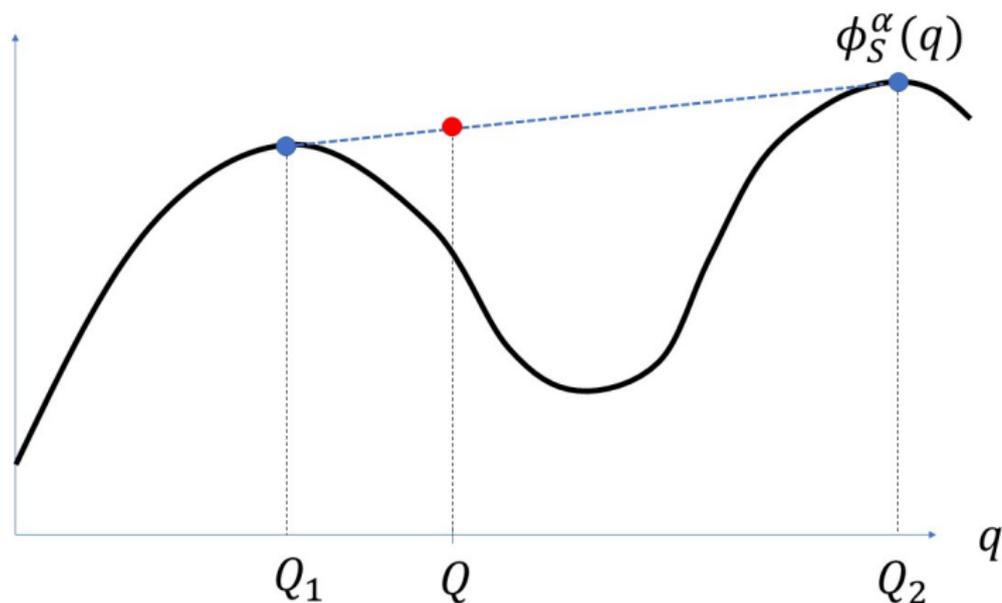
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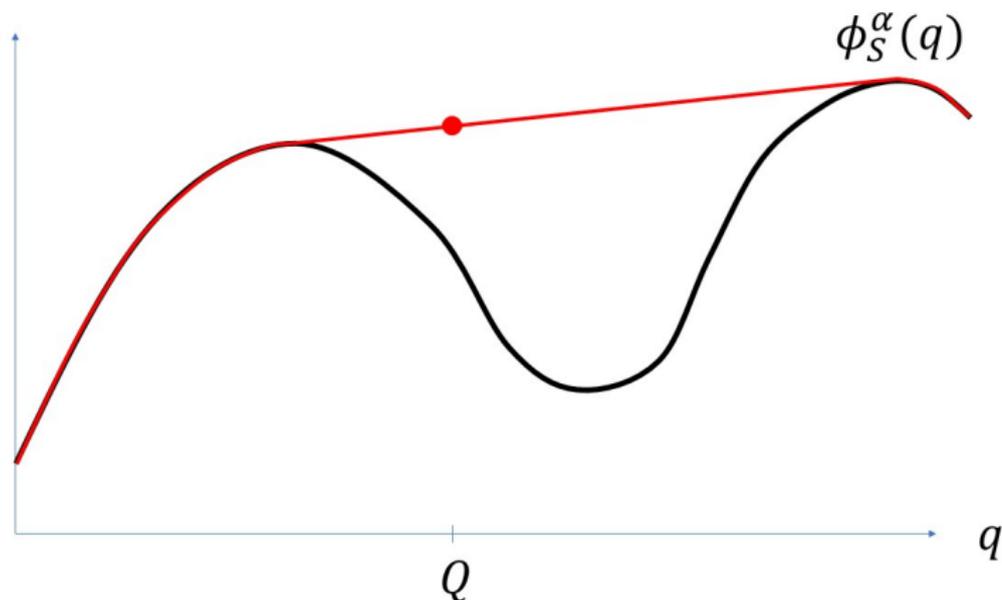
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Bonus Step: Linear Algebra eliminates some constraints

$\rightsquigarrow$  four-price characterization

# What if direct redistribution is not feasible?

Lump-sum transfers give agents a constant amount of money even if they do not trade.

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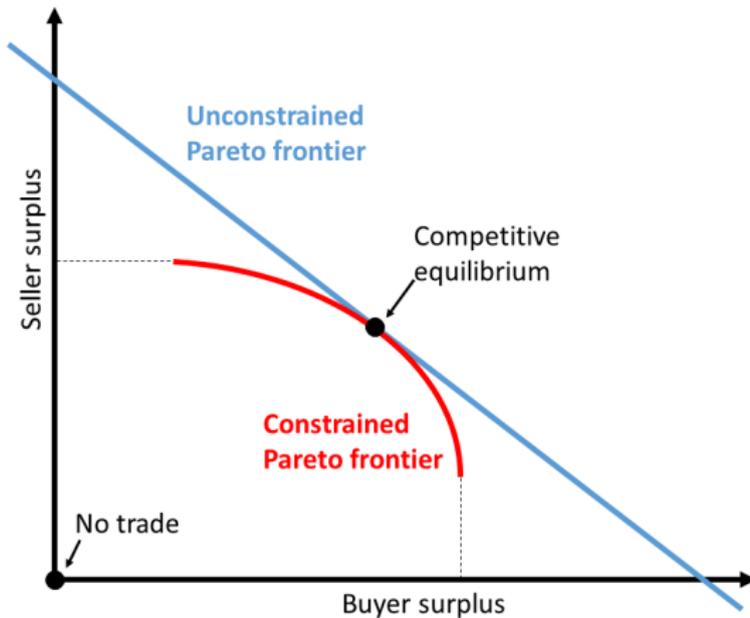
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↪ Rationing can emerge even if there is no same-side inequality (provided that cross-side inequality is sufficiently large).

# Model – Welfare



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- Intuition?
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- Simple Mechanisms
- Optimal Mechanism
  - Proof Sketch
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- Wrap (wrapped)